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A multi-country prosperity index by two-dimension singular spectrum analysis

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Abstract

With the development of the global economy, interaction among different economic entities from one region is intensifying, which makes it significant to consider such interaction when constructing composite index for each country from one region. Recent advances in signal extraction algorithm and time series modeling technique makes such consideration not only feasible but also practical. Singular spectrum analysis (SSA) is a newly developed tool for time series modeling and proven to be a powerful tool in extracting signals. In this paper, this method is introduced into multi-country business cycle analysis for the first time. MSSA is employed to construct a model-based composite index and the 2D-SSA is used to establish the multi-country composite index. Empirical results performed on Chinese economy demonstrate the accuracy and stability of MSSA-based composite index, and the 2D-SSA based composite indexes for Asia countries show their efficiency in capturing the interaction among different countries.
Keywords: Singular Spectrum Analysis; 2D-SSA; Business cycle analysis; Multi-country composite index.

1. Introduction

The recent global economic recession rooted in the US subprime crisis and the quick recovery of China and other emerging countries causes both the experts and laymen alike to wonder what is happening and will happen to the word economy. Marcellino (2010) mentioned that if one person had fallen asleep in 2007 and woken up in 2009, she would ask what happened to the world economy. The answer is that the business cycle is alive and kicking.

Business cycle monitoring and forecasting has always been the focus of economic analysis, and composite index is the commonly used tool. The latest global economic crisis stimulates a renewed interest in studying the prosperity index, partly due to both the current status of the global economy and a set of new theoretical developments as well. The prosperity index could be traced back to the application of “Harvard index” to monitoring United States business cycle in early 20th century. Although the Harvard index is not very successful in predicting business cycle, it does stimulate great interest in index research. Following the idea of Harvard index, the National Bureau of Economic Research (NBER) of the US developed diffusion index and composite index . Theoretical advancements in econometrics promote the development of model-based index, which is considered to be a breakthrough for business cycle research. Marcellino (2006) classified the model based indices into two categories, one of which is factor based model. Factor based models think the economic systems are driven by some common factors behind a

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pool of economic variables, and attempt to construct the index out of the common factors. A typical factor based index is the SW index proposed by Stock and Watson (1989, 1991, 2002). This index extracts a limited number of common factors from a set of variables, attempting to provide a formal probabilistic basis for Burns and Mitchell's coincident and leading indicators. However, SW's procedure requires an ex-ante classification of variables, in order to extend it into large datasets, Forni, et al., (2000, 2004) proposed an alternative factor based methodology that addresses both issues, and applied it to the derivation of a composite coincident indicator for the Euro.

Most of the existing theoretical literatures focus on individual economy, however, one of the most important issues in business cycle analysis is how to construct a multi-country prosperity index. It is impossible to monitor and compare the performance of each country without constructing the multi-country prosperity index. It has been well documented in many literatures, for example, Artis, et al., (1997, 2004,) analysis business cycle for European Countries based on industrial production. Fidrmuc and Korhonen (2006) reviewed the literatures on business cycle, and analysis the business cycle correlation between the newest EU members and the Central and Eastern European countries based on 35 identified publications. Forni, et al., (2001) established a coincident and a leading indicator for EURO area with dynamic factor analysis, and so on. Many institutions also construct their own composite index for each country to monitor the economic development, like OECD, IMF, EC and so on.

Economic interaction among different countries has been well documented in huge literatures; however, studies, either theoretical or empirical, successfully capturing such interaction are still far less accessible in business cycle analysis. In order to compare the performance of different countries in one region, many studies consider the interaction by introducing several indicators. Most of them construct the composite index for each country based on similar approach, and then compare them. Things may change with the recent advances in time series analysis. A newly developed tool, known as singular spectrum analysis, can solve this problem by including the interactive influence of other countries in the composite index. 2D-SSA approach can successfully take advantage of the interactive influence and construct the multi-country composite index at the same time. On one hand, SSA approach can be used as a model based method to construct the composite index, just like most of the previous studies in Forni, et al., (2000), Stock and Watson (2002). On the other hand, 2D-SSA could be employed to construct the multi-country composite index simultaneously and take the interaction among different countries into account.

In this paper, we introduce the singular spectrum analysis into the business cycle analysis for the first time and test its performance. The paper organized as follows. In Section 2 we describe the singular spectrum analysis method for the construction of composite indexes. In Section 3, we use multi-dimension time series SSA to construct the composite index and compare its performance of traditional technique. Finally, we use the 2D-SSA to construct the multi-country composite index.

2. Singular spectrum analysis method

Singular spectrum analysis (SSA) is a newly developed nonparametric tool for modeling time series and extracting signals, requiring no prior knowledge of the information with the model. The idea behind the SSA is very intuitive. It attempts to decompose the original time series into the sum of a small number of independent and interpretable components such as a slowly varying trend, oscillatory components and noise. Theoretical and practical foundations of the SSA technique are available in Golyandina, Nekrutkin, and Zhigljavsky (2001), and an elementary introduction to the subject can be found in Elsner and Tsonis (1996).SSA has a wide range of applications: from meteorology and physics to economics and financial mathematics. SSA was first applied to extract tendencies and harmonic components in meteorological and geophysical time series (Vautard, Yiou, and Ghil, 1992). In recent years SSA has been developed and applied to many practical problems, and empirical results demonstrate the great efficiency of SSA. Multichannel SSA (MSSA) is intended to analyze simultaneously a set of time series with common features (Elsner and Tsonis, 1996; Golyandina and Stepanov, 2005). MSSA can be applied to 2D scalar fields if one dimension is considered as time. The 2D-SSA is specially designed to process 2D scalar fields at first (N. Golyandina, K. Usevich (2010,)) and has been used for image processing. If we parallelize the multi-country data, we may draw a three dimension (country~index~time) in a picture, and then use 2D-SSA to deal with it. Selecting different window width, the fluctuation and contagion among different countries will be taken into account.

As a powerful tool for time series modeling and signal extracting, SSA is introduced into business cycle analysis, which is the first attempt to use SSA method in business cycle analysis.

2.1. Univariate singular spectrum analysis method

The SSA is becoming more and more popular, especially in applications. The fundamental idea of SSA is to decompose the original time series into a sum of a small number of interpretable components identified as trend, periodical components, and noise. SSA is based on the singular-value decomposition of a specific matrix constructed from time series, which makes it a nice model-free technique. SSA technique consists of two stages: decomposition and reconstruction and both of which contain two separate steps. Following the way of Golyandina (2001) and Hassani (2007), we present the discussion on the SSA technique as follows.

Stage 1. Decomposition

1st step: embedding.

Embedding can be regarded as expanding the original time series into a trajectory matrix which is related to a certain window length known as the embedding dimension. For a given time series X={x_1, x_2,…, x_n}, expanding X yields the following trajectory matrix

$$Y=\left\{\begin{matrix} x_1 & x_2 & \cdots & x_m \\ x_2 & x_3 & \cdots & x_{m+1} \\ \bullet & \bullet & \bullet & \bullet \\ x_{n-m+1} & x_{n-m+2} & \cdots & x_n \end{matrix}\right\},$$

where m is the window length (embedding dimension). Window length should be large enough to capture the global behavior of the original system. The trajectory matrix Y obtained is a Hankel matrix, which means that all elements along the diagonal i+j=const are equal. Window length is the only single parameter of embedding.

2nd step: singular value decomposition.

Singular value decomposition performed on the trajectory matrix Y yields the following equation,

$$Y=USV^T \tag{1}$$

where U is a (n-m+1)×(n-m+1) real orthogonal matrix, V is a m×m real orthogonal matrix and S is a (n-m+1)×m diagonal real matrix whose elements are the singular values of the trajectory matrix Y. S can be obtained by computing the following matrix:

$$C=Y^TY \tag{2}$$

Since C is a symmetric semidefinite positive matrix, there exists a unique spectral decomposition equation:

$$C=\Phi\Lambda\Phi^T \tag{3}$$

where Φ is a real orthogonal matrix with its columns the eigenvectors of C, and Λ is a real diagonal matrix such that its elements σ_i^2 are the eigenvalues of C in decreasing order. It is easy to obtain that:

$$C=Y^TY=(USV^T)^T(USV^T)=VS^TU^TUSV^T$$

Since $U^TU=I$ and $S^TS=S^2$, one obtains equation:

$$C=VS^2V^T \tag{4}$$

It is obvious that

$$\begin{matrix}\Phi=V \\ \Lambda=S^2\end{matrix}, S=\left\{\begin{matrix}\Lambda \\ \mathrm{O}\end{matrix}\right\}.$$

Where $\Lambda=diag\{\sigma_1,\sigma_2,...,\sigma_m\}$, O is a null matrix with all elements equal 0.

Equ. (1) can also be written as equation 5

$$Y=\sum_{i=1}^{m}\sigma_i u_i v_i^T \tag{5}$$

where u_i and v_i are respectively U's and V's matrix i-th columns. Equ. (5) is called spectral decomposition. Given $V=\Phi$ and $YV=US$, it is obvious that $v_i=\Phi_i$ and $\sigma_i u_i=Y\Phi_i$ where Φ_i is i-th column, which allows to obtain a simpler and more useful form for Y's decomposition:

$$Y=\sum_{i=1}^{m}Y\Phi_i\Phi_i^T \tag{6}$$

Stage 2. Reconstruction

1st step: Grouping

The grouping step can be seen as separating the time series into two components: "the signal" and the "noise". The idea is to project the trajectory matrix on a q-dimensional space:

$$Y_q = \sum_{i=1}^{q} \sigma_i u_i v_i^T \tag{7}$$

And

$$noise = \sum_{i=q+1}^{m} \sigma_i u_i v_i^T \tag{8}$$

q is the only parameter in the grouping. The first q leading eigentriples are associated to the signal and the remaining (m-q) eigentriples are associated to the noise.

2nd step: diagonal averaging

The purpose of diagonal averaging is to transform a matrix to the form of a Hankel matrix which can be subsequently converted to a time series. The diagonal averaging procedure is clear by looking at the following example.

For example, if Y_q is the matrix:

$$Y_q = \begin{Bmatrix} x_{1,1} & x_{1,2} & x_{1,3} & x_{1,4} \\ x_{2,1} & x_{2,2} & x_{2,3} & x_{2,4} \\ x_{3,1} & x_{3,2} & x_{3,3} & x_{3,4} \\ x_{4,1} & x_{4,2} & x_{4,3} & x_{4,4} \end{Bmatrix}$$

Taking diagonal averaging to Y_q yields the following time series

$$X_q = \left\{x_{1,1}, (x_{2,1} + x_{1,2})/2, (x_{4,1} + x_{3,2} + x_{2,3})/3, (x_{4,2} + x_{3,3})/2, x_{4,3}\right\}$$

As for the construction of non model based composite indexes, Marcellino (2006) mentioned that mainly three elements, the selection of the index components, the transformation of the index components and construction of a composite index is the choice of a weighting scheme. From the technique aspect, the most of important of them is the transformation of the index components, and there are many literatures related to the choice of the procedure for the transformation of the index components.

SSA is a newly developed tool for data processing, it can deal with data with missing data[a] and it is also very powerful to detect the turning point, see , for example , Moskvina and Zhigljavsky (2003), As a result, SSA will be a very powerful tool for the construction of non model based composite index.

2.2. Multivariate singular spectrum analysis method

Multivariate SSA is an extension of the standard SSA to the case of multivariate time series. Assume we have two time series $X_T = \{x_1, x_2, ..., x_T\}$ and $Y_T = \{y_1, y_2, ..., y_T\}$, and let L be the window length. Using embedding terminology, we can define the trajectory matrices M_X and M_Y obtained from time series X_T and Y_T respectively. The trajectory matrix can then be defined as:

$$M = \begin{pmatrix} M_X \\ M_Y \end{pmatrix}$$

The other stages of the multivariate SSA procedure are identical to the univariate SSA. The generation to the case of several series is straight forward.

Marcellino (2006) also pointed out that there are two main methodologies within the model based approaches for the construction of a composite coincident index (CCI). They are dynamic factor models and Markov switching models. In both cases there is a single unobservable force underlying the current status of the economy, but in the former approach this is a continuous variable, while in the latter it is a discrete variable that evolves according to a Markov chain.

In this paper, we will introduce another model based approach for the construction of the composite coincident

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a There are different SSA-based methods for filling in missing data sets (see, for example, Schoellhamer, 2001; Kondrashov et, al., 2005) index. Multivariate SSA can be used as a model based approach.

Firstly of all, a group of indicators that related to the reference indicator should be selected carefully. Then grouping them into $x_{Nt} = \{x_{1t},...,x_{Nt}\}^T$, and SSA does not required any prior knowledge of the data series and does not require the stationary of the data as well. Finally, using the MSSA to analysis the data, a filtered data group will be available after reconstruction. And the filtered series related to the reference indicator will be a composite index. This index is the common factors of all the data.

2.3. Two-dimension singular spectrum analysis method

The 2D-SSA was specially designed to process 2D scalar fields. Unlike MSSA, 2D-SSA is invariant regarding field rotation. 2D-SSA has been used for image processing. However, we will show that it is also very powerful for multi-dimension time series analysis.

When analyzing multi-countries data series, for a single indicator, we will get a matrix $x_{Nt} = \{x_{1t},...,x_{Nt}\}$, where N denotes N countries and x_{it} denotes that indicator of country i. However, when we come across several indicators of multi-countries, and we want to get a composite index for each country, model based approach or non-model based approach are usually employed for each country respectivelyb. Therefore, the interactive influence of multi-country is usually ignored and the comparisons among the multi-country are also not based on the same criterion.

Two-dimension singular spectrum analysis makes it possible for analysis multi-country data simultaneously.

Let us consider a 2D data F: $\{X_{11},....,X_{mn}\}\times\{X_1,...,X_t\}$, where X_t is one indicator, and m represents that there are totally m index for each country and n denotes that there are n countries in all.

The other stages of the 2D-SSA procedure are close to the univariate SSA, and for model detail information, see to Golyandina (2010).

There is an important difference between 2D-SSA and SSA when choosing the parameters. Window length plays a key role in decomposition, and it is a value (L_1) in SSA but two numbers (L_1, L_2) in 2D-SSA. One is also similar to SSA, which is selected by the properties of the X_t in data F. When L_2 equal to 1, it is a special case which is equivalent to using SSA. When n L_2 equal to m, which is the number of the index, other country's information will be reflect in one's composite index. When n L_2 equal to mn, it will be equivalent to using MSSA. And only one country's own information can be used. We will show this characteristic of 2D-SSA in the next section for detail.

3. Prosperity index by SSA

As a powerful data filtering tool, SSA can be used to construct a non-model based composite index: filtering data with SSA, and then compositing the filtered data based on weight. It is just a simple case of the application of SSA. In this paper, we'd like to focus on the performance of SSA used as a model based approach for prosperity index. Basically, SSA is a factor based method, Similar to other factor based model. MSSA can be used to construct a model based composite index. Further, in order to analysis multi-country in one region, 2D-SSA is used to construct a multi-country composite index, which considers the interaction among different countries at the same time.

3.1. Model based composite index using MSSA

In this section, we will use MSSA to analyze China's business cycle. Since our work is based on the growth rate cycle, all of data are growth rate. Because the monthly data of GDP is not available, value added of industry is considered as the reference indicator.

In order to compare the property of the composite index based on MSSA, we choose another composite index based on X12-ARIMA which in the project cooperated with Center bank in China.

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b See, for example the global composite indicator of OECD, IMF and other institution.

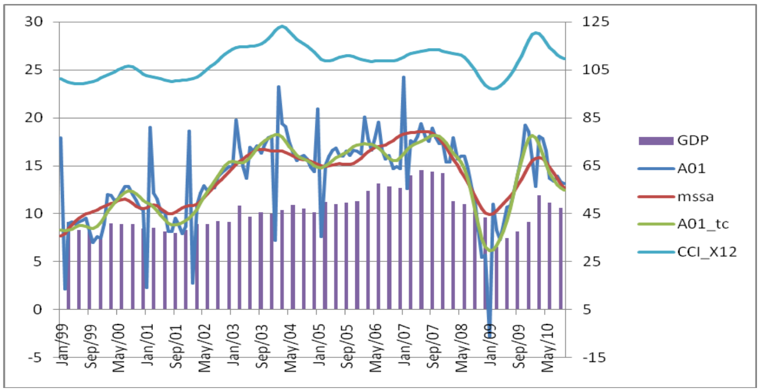

Fig. 1 The comparison of MSSA and non-model based indexc

Fig.1 presents the plots of MSSA and non-model based index. MSSA is able to deal with a large group of data set, the prosperity index based on MSSA is much more stable than non-model based index. For example, during the period of 2005-2007, there are several fluctuations in the data of non-model based index while MSSA composite index keep stable in this period.

In business cycle analysis, prosperity index is established to reflect the overall economic performance. Typically, GDP data is selected as reference data, because it is the most efficient variable describing the economic system. Because of the inaccessibility to monthly data for GDP, many literatures use industry value added index as a substitution to be get a timely index. Although, industry value added is a good substitution, it may act inconsistent with GDP data when describing the turning point, the fluctuation extent, etc. Thus, in order to get a better prosperity index, it is necessary to compare with GDP data when value added of industry is used as a reference data. However, most of composite index and their reference series have similar trend and turning points, see Figure 1. The advantage of MSSA based composite index is that the value of MSSA-based composite index makes more sense than non-model based one. The performance of the MSSA-based composite index is much more consistent with the overall economy. For example, the peak in May, 2004 of non-model based index is higher than the one in Sep, 2007, though it is coincident with the industry value added growth rate, and it goes against the GDP growth rate. The value of GDP growth rate in Sep. 2007 is much higher than that in May, 2004. Although industry value added index is used as a proxy variable for GDP for the consideration of data accessibility since there are no monthly observations for GDP, industry value added is not a very good proxy sometimes. In order to analyze the economic growth, it is necessary to compare with GDP data sometimes. MSSA based prosperity index demonstrates a more stable and significant result.

3.2. Multi-country composite index based on 2d-SSA

With the development of global economy, there is growing interest concerning the multi-country composite index. For example, Forni, et, al. (2004) uses FHLR method to construct a regional composite index for European. Some institutions, like OECD, IFO, IMF and so on, also have their own composite index for many countries. So far, these composite indexes still focus on the comparison of difference countries. They use the same approach to construct the composite index for different countries and then compare them. Although the interactive influence among different countries have gained a lot of attention in many fields, like finance, macroeconomic, trade and so on, it is still not taken into consideration in the construction of composite index in business cycle analysis.

In section 3.1, it has been demonstrated that the composite index based on SSA approach is more stable and sensible compared with other methods. Further, SSA approach is also able to get some results which cannot be obtained by other methods. We will illustrate as follows the construction of multi-country composite index based on

c A01 denotes Value Added of Industry, A01-TC denotes the trend and cycle component of Value Added of Industry based on X12-ARIMA, mssa denotes the composite index based on MSSA method, GDP denotes the Gross Domestic Product.CCI_X12denotes the composite index based on X12-ARIMA method and traditional OECD method.

2D-SSA using a sample of 7 countriesd in Asia.

In order to compare the performance and monitor the economic growth in these countries, similar indicators of each country are employed covering economic growth, finance, price, trade, industry and so on. There are seven indicators for each country. Fig. 3 presents the plots of input data in the left panel, output data in the middle panel and residual in the right panel.

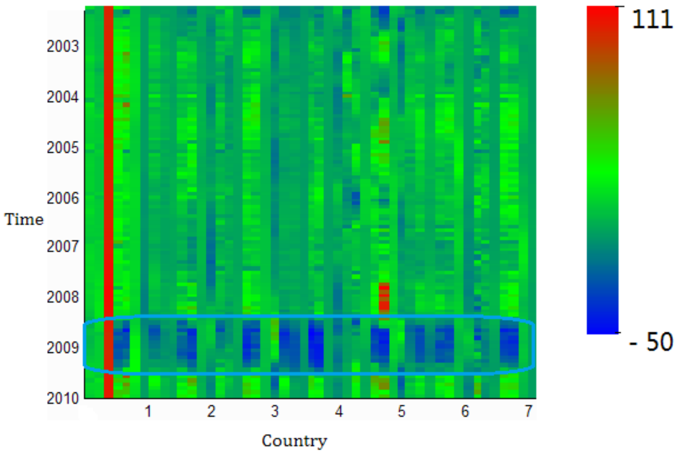

Fig. 2 Input data of Asia countries

Fig. 2 presents the plot of input data of Asian countries. The fluctuations of each indicator in each country are quite clear in the input data plot in Fig. 2; countries from one region have similar economic structures; the relations among the indicators which denote different respects of the economy are also similar as well. It can also be obtained from Fig. 2 that the data can be clearly divided into seven parts according to the similar data structure of these seven countries. Further, the interaction among these countries are also vivid, there is a sharp decline for all these countries in the period of financial crisis, see the input data structure in the blue box. Methods, constructing composite index separately and then comparing them, fails to utilize such data structure. The multi-country composite index to be constructed can make good use of this data structure and interaction.

2D-SSA is used to construct the composite index. It has been illustrated that one important parameter in the method of 2D-SSA is the horizontal window length selection. The horizontal window length determines the window width in the horizon when decomposing and reconstructing. For example, if the window width is large enough, all the data will be considered at the same time, otherwise, only less data will be included. Multi-country composite index will be constructed utilizing this feature of 2D-SSA.

First, when horizontal window length L2 is chosen to be 49 (7×7), the 2D-SSA performs much like MSSA. The output plot in Fig. 3 is the filtered data for the overall indicators. Fig. 3 presents the composite index for each country selected from the final output data in Fig. 3.

Focusing on individual country's economy, Fig. 3 presents the plots of prosperity index of each country constructed by 2D-SSA. It can be observed from Fig. 2 that due to the fluctuation of data of individual country, the index are not quite stable. For example, the prosperity index of Thailand at the bottom panel in Fig. 3 indicates extremely different performance compared with the other countries. During the Asian financial crisis, Thailand experience a sharp decline in economy growth, thus during the next several years, Thailand gains a much higher economic growth. That's why Thailand experienced a very high peak in 2004-2005. If the interaction among countries in one region is taken into consideration, the results will be better.

Hence, the window length is chosen to be 7 for the consideration of interactive influence among different countries. Fig. 4 presents the plots of composite index of each country. With the fast development of the global economy, the interactions among different countries are intensified. To capture such interaction, the most commonly

d These Seven countries are China, Japan, Korea, Indonesia, Malaysia, Singapore, Thailand.

employed technique is to introduce some indicators reflecting global influence. However, the composite index based on 2D-SSA could well reflect other countries information without introducing other new indicators. When the window length is equal to 7, the 2D-SSA procedure can filter out the similar fluctuation among different countries. Thus, the information of one country will also be available for other country.

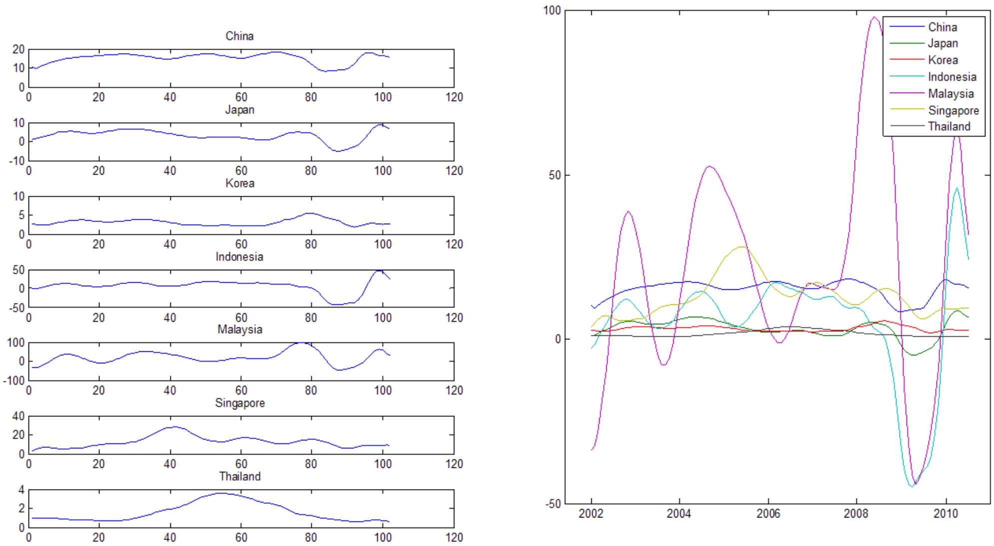

Fig. 3 2d-SSA based Composite Index (width=49)

Fig. 4 presents the composite index for each country constructed by 2D-SSA. The dominance of composite index in Fig. 4 over that in the previous one is obvious. For example, during the financial crisis (2009-2010), the latter composite index performs much better than the former one does, which makes the comparison among different countries meaningful from economic perspective.

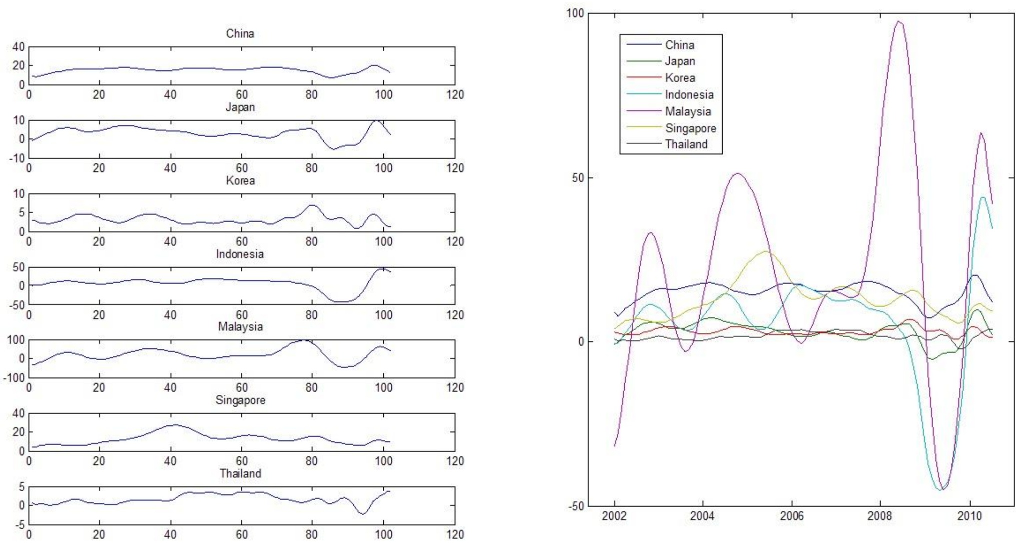

Fig. 4 2d-SSA based Composite Index (width=7)

4. Conclusion

This paper is the first attempt introducing SSA to business cycle analsyis. MSSA is employed to analyze individual country's business cycle and we take its advantage to analyze China's business cycle in this paper. Although regional composite index has gained a lot of attention, theoretical advances are still far less available. In this paper, we introduce 2D-SSA approach, which can be a specialized tool for the construction of multi-country composite index. Empirical results demonstrate the accuracy and stability of the SSA approaches in constructing composite index, which makes it a nice tool for analyzing not only economic business cycle but also other cycles like financial business cycle, industrial business cycle, and so on. Some issues remain unsolved and need further studies. For example, many previous literatures have shown the powerful ability of SSA in prediction. The powerful forecasting ability of SSA makes it a potential for constructing the leading index, which will be the focus of our future studies.

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